On the behaviour of small disturbances in plane Couette flow

Part 2. The higher eigenvalues

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(Received 12 August 1963)

In an earlier paper (Gallagher & Mercer 1962) the numerical results for the first eigenvalue of the problem of plane Couette flow were given. The higher eigenvalues are now examined and are found to be in agreement with those of Southwell & Chitty (1930), but to disagree with those of Grohne (1954).

In an earlier paper (Gallagher & Mercer 1962), the authors gave the results of their calculations on the first eigenvalue for the problem of small disturbances in plane Couette flow of a viscous incompressible fluid. The equations governing the disturbance were linearized and on the assumption that the velocity, for example, in the transverse direction, could be obtained from a superposition of functions of the form $v(y)\exp\{i\alpha(x-\xi t)\}$, the admissible values of ξ were calculated from the differential equation

$$v^{\text{iv}}-2\alpha^2 v''+\alpha^4 v-i\alpha R(y-\xi)\,(v''-\alpha^2 v) = 0.$$

In the earlier paper only the results for the first eigenvalue were given, good agreement being obtained with those of Hopf (1914), Southwell & Chitty (1930), Grohne (1954) and in a more recent paper, Deardorff (1963) confirmed these results. Since then the higher eigenvalues have been analysed and satisfactory agreement has been obtained with the results of Southwell & Chitty (Fig. 6, p. 248). Figure 1 shows this comparison in terms of λ', where $\lambda' = 4\alpha(iR\xi-\alpha)/\pi^2$ (Gallagher & Mercer 1962, p. 94), for the first four eigenvalues for the case $\alpha = 2$.

The present results for the higher eigenvalues, however, do not agree with those of Grohne, as is shown by a comparison of figure 2 with figure 3. In the former diagram the present results for the imaginary part of ξ have been plotted against R for the case $\alpha = 1$, while Grohne's results are given in the latter. It will be noticed that these diagrams show considerable differences both in the general trend of the eigenvalues and in the values of R at which two real eigenvalues join together to form a complex conjugate pair. It has not, however, been possible to explain the reason for this disagreement.

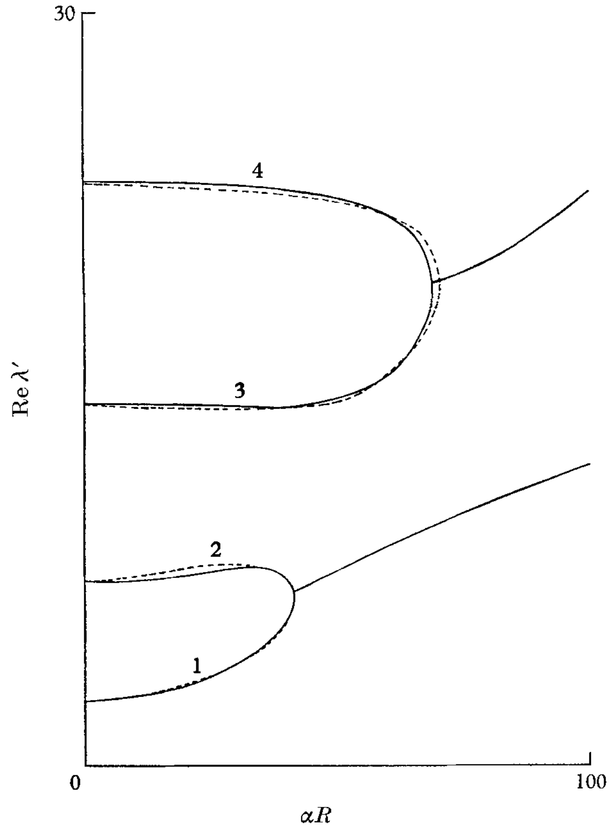

FIGURE 1. Variation of the first four eigenvalues $\mathrm{Re}\,\lambda'$ with αR for the case $\alpha = 2$; – – –, values obtained by Southwell & Chitty.

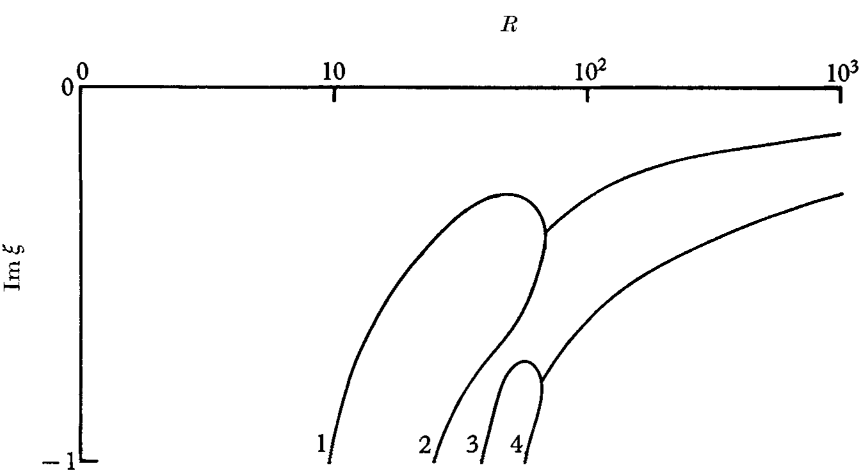

FIGURE 2. Variation of the first four eigenvalues $\mathrm{Im}\,\xi$ with R for the case $\alpha = 1$, as obtained by the present results.

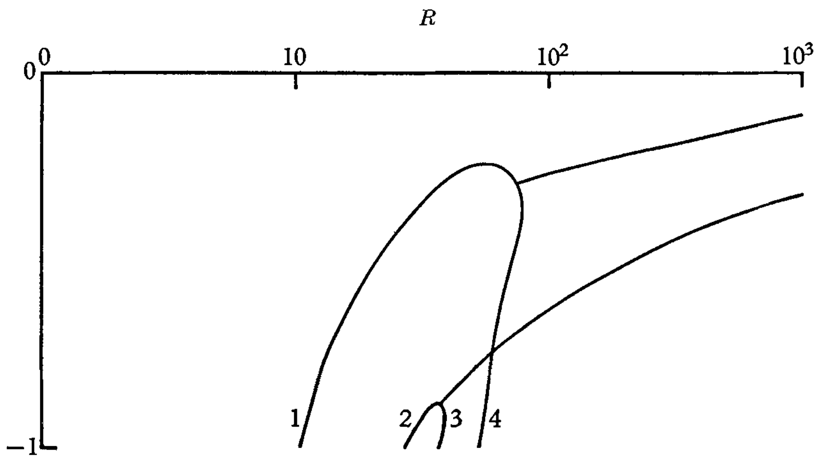

FIGURE 3. Variation of the first four eigenvalues Im ξ with R for the case $\alpha = 1$, as obtained by Grohne.

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